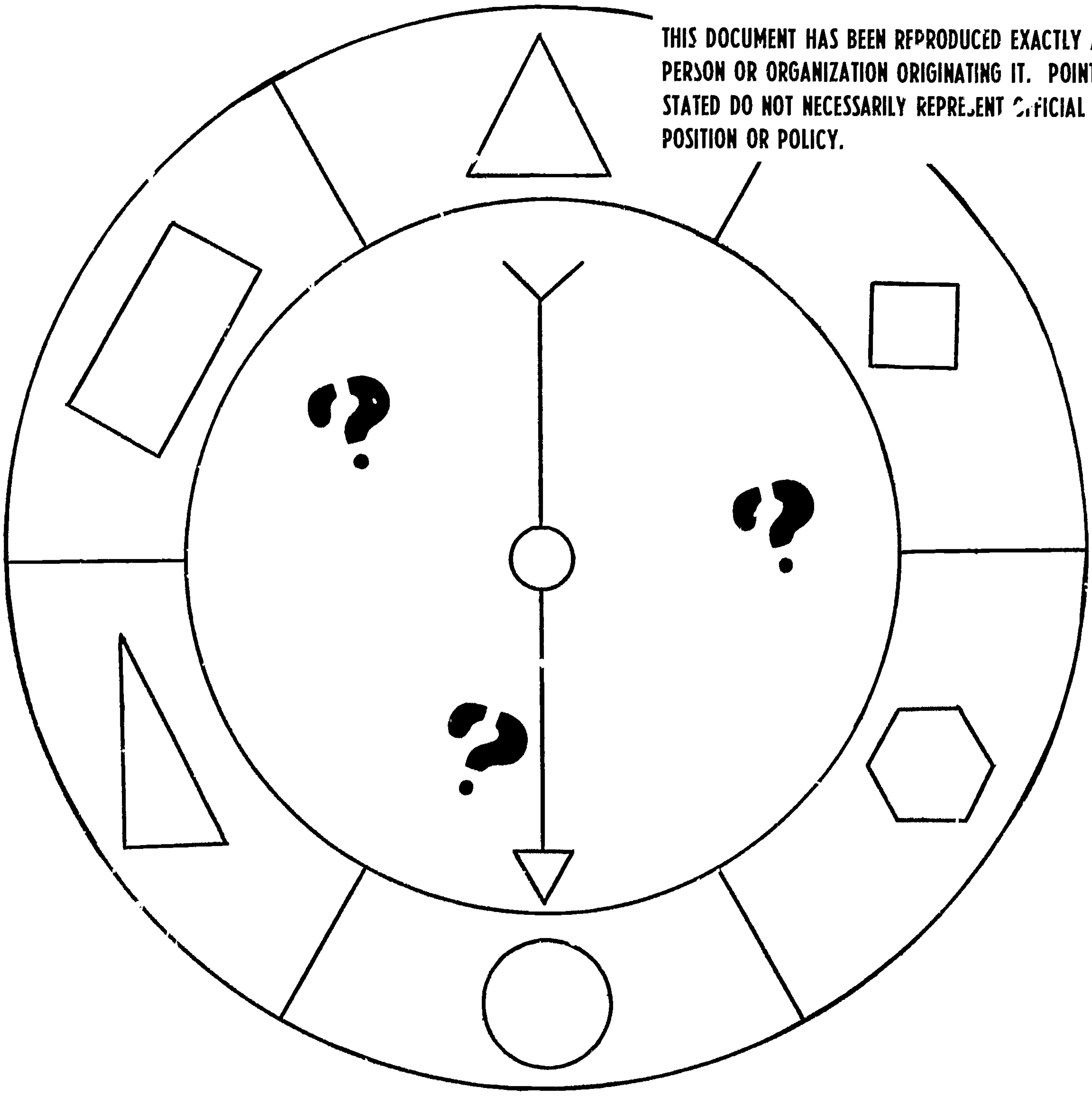
DOCUMENT RESUME

ED 021 732 SE 004 523

By- Foley, Jack L.
EVENTS AND CHANCE.
Pub Date Aug 67
Note- 15p.
EDRS Price MF-$0.25 HC-$0.68
Descriptors- ARITHMETIC, CURRICULUM, *CURRICULUM DEVELOPMENT, *ELEMENTARY SCHOOL MATHEMATICS, *INSTRUCTIONAL MATERIALS, LOW ABILITY STUDENTS, MATHEMATICS, *PROBABILITY, *SECONDARY SCHOOL MATHEMATICS, STATISTICS
Identifiers- Elementary and Secondary Education Act Title III

This booklet, one of a series, has been developed for the project, A Program for Mathematically Underdeveloped Pupils. A project team, including inservice teachers, is being used to write and develop the materials for this program. The materials developed in this booklet include (1) the meaning of probability, (2) counting outcomes, (3) mutually exclusive outcomes, and (4) independent outcomes. (RP)



EVENTS & CHANCE

ESEA Title III

PROJECT MATHEMATICS

Project Team

Dr. Jack L. Foley, Director
Elizabeth Basten, Administrative Assistant
Ruth Bower, Assistant Coordinator
Wayne Jacobs, Assistant Coordinator
Gerald Burke, Assistant Coordinator
Leroy B. Smith, Mathematics Coordinator for Palm Beach County

Graduate and Student Assistants

Jean Cruise
Kathleen Whittier
Jeanne Hullihan
Barbara Miller
Larry Hood

Donnie Anderson
Connie Speaker
Ambie Vought
Dale McClung

Secretaries

Novis Kay Smith
Dianah Hills
Juanita Wyne

TEACHERS

Sister Margaret Arthur
Mr. John Atkins, Jr.
Mr. Lawrence Bernier
Mr. Harry Berryman
Mr. Ricke Brown
Mrs. Nicola Corbin
Mrs. Gertrude Dixon
Mrs. Dellah Evans
Mrs. Marilyn Floyd
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Mrs. Aleen Harris
Mr. Earl I. Hawk
Mr. Arthur Herd
Mrs. Alice Houlihan
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Mr. Charles G. Owen
Mrs. Margaret Patterson
Sister Ann Richard
Mr. Carl Sandifer
Mrs. Elizabeth Staley
Mr. James Stone
Mrs. Linda G. Teer
Mr. James Wadlington
Mrs. Marie Wells
Mr. Ronald Whitehead
Mrs. Mattie Whitfield
Mr. James Williams
Mr. Kelly Williams
Mr. Lloyd Williams

August, 1967

For information write: Dr. Jack L. Foley, Director
Bldg. S-503, School Annex
6th Street North
West Palm Beach, Florida

EVENTS AND CHANCE

Have you ever made any of the following statements?

Chances are, we will win this game.

We have a good chance of winning.

This game is a tossup.

Chances are, it will rain.

He probably will not live to be 100 years old.

The odds are in our favor.

In many cases a number can be used in connection with chance statements. Each of us has made "chance" statements. Did you know that you are actually asking questions or making statements that involve one of the most interesting applied areas of mathematics? This part of mathematics is called probability.

Probability is a relatively new area of mathematics as compared to many other areas. Historically, probability originated in connection with gambling. The first published report, dated 1494, involved two gamblers. The two men were to play for a given number of points, but they were forced to stop before the game was completed. Each man had a different score. The question was: What portion of the stakes did each man deserve?

Almost 140 years passed before this question was answered. In the 17th century, it was solved by two different people using different procedures. The two men were Pascal and Fermat. Correspondence between these two mathematicians led to the development of probability.

For many years after the development of probability, it was applied mainly to the area of gambling. Today, however, it has almost unbelievable applications in many different areas. More applications are constantly being found and more aspects of probability are being developed.

Suppose we look at some of the uses of probability in our society.

Insurance	--	To determine insurance rates
Major Industries	--	For production control and possible sales
Medical Profession	--	In research, treatment, and the prescription of medicines

Military	--	Strategies of battles
Government	--	Tax programs, spending, and many other areas
Large-Scale Farmers	--	To plan and harvest crops
Educators	--	To plan facilities, programs, and advise students
Cities	--	To plan building programs, civic activities, police and fire protection, and many other areas
Politicians	--	To plan election campaigns
Advertising	--	To develop effective advertising plans
Gambling Establishments	--	Guess why?

These are only a few examples, but can you imagine that this vast area of mathematics could arise as a result of an apparently simple gambling question? Suppose we look at some basic probability ideas.

Consider the following experiment:

A class of thirty students selects its class president in the following way. The name of each student is written on a slip of paper and placed in a box. The slips are mixed up by shaking the box. Without looking, a person draws a slip. The person named on the slip is appointed class president. Discuss the following questions.

1. If drawing a particular student's name is one outcome, how many different outcomes are possible?

2. Do you have a chance to be class president? What are your chances?

In your answer to the first question, did you agree that there are thirty possible outcomes?

The second question is concerned with exactly one outcome.

Then we say that the probability of your being president is:

$$\text{Probability (of your being president)} = \frac{1}{30}$$

Notice that the denominator of the probability is the total number of possible outcomes. The numerator is the number of favorable outcomes or the specific outcomes of interest.

Suppose you are interested in the probability of your <u>not</u> being class president, or simply the probability that someone, other than you, <u>will be</u> class president. Then, how many of the <u>total possible</u> outcomes would satisfy this condition? That is, how many slips of paper do <u>not</u> have your name on them?

That's correct; there are <u>29</u>. Then:

$$\text{Probability (of someone else as president)} = \frac{29}{30}$$

This brings up two interesting questions:

A. What is the probability that you <u>or</u> someone else will be class president? This is what we call a <u>certainty</u>. What is the number value we give a certainty? We have

$$\text{P (of a class president)} = \frac{30}{30} = \underline{1}$$

B. Suppose someone asks, "What is the probability that a person, whose name is <u>not</u> on one of the slips, will be class president?" Then:

$$\text{P (of a person whose name is not on a slip)} = \frac{0}{30} = \underline{0}$$

Then if an event has a probability measure of <u>0</u>, it <u>cannot</u> occur.

From this we can see that a probability will either have a measure <u>between</u> 0 and 1, or have a measure of <u>0</u> <u>or</u> <u>1</u>.

In our last example, the particular outcome was picking a slip with your name on it. A particular outcome of an experiment is called an <u>event</u>.

From our discussion to this point, we can now list some basic ideas in determining a probability.

<u>IDEA I</u> - We need to determine the <u>total number</u> of different outcomes that result from an experiment.

<u>IDEA II</u> - Determine the <u>number of specific outcomes</u> that we are interested in and simply call them "specific outcomes."

Then:

$$\text{Probability (of specific outcomes)} = \frac{\text{Number of Specific Outcomes}}{\text{Total Number of Outcomes}}$$

To illustrate these two ideas, consider the following experiment:

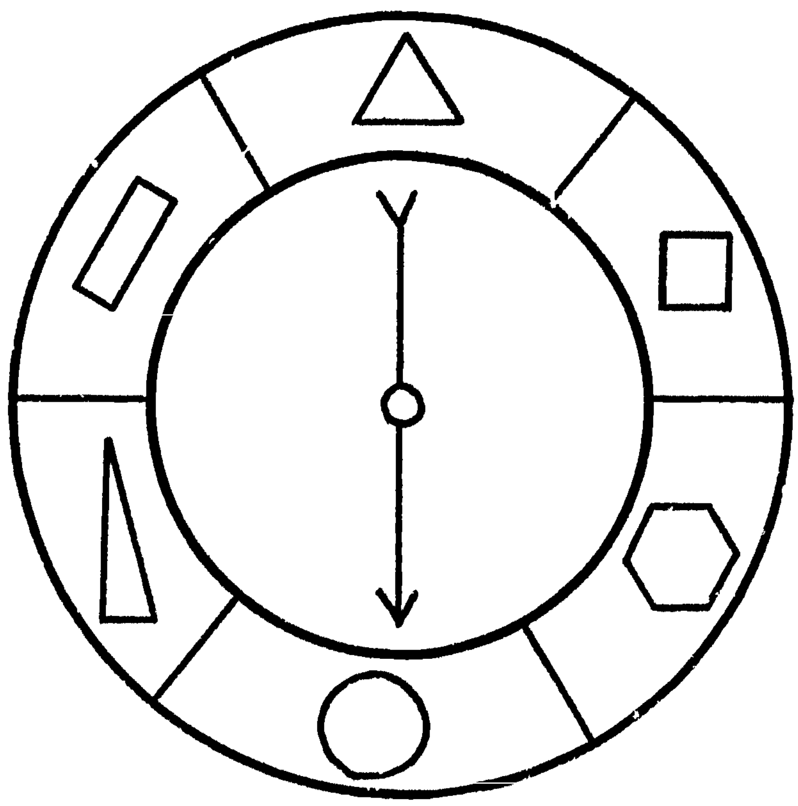
Four selection buttons appear on a soft drink machine. The choices are coke, orange, grape, and pepsi. However, the name of the drink, corresponding to the selection button, has been removed. You put in a dime and push a button. What is the probability of getting a pepsi? (Assume the machine works.)

IDEA I - Four possible outcomes.

IDEA II - One specific outcome of interest.

Then: Probability (of pepsi) $= \frac{1}{4}$

See if you can apply these ideas to solve the following problems. Each problem is based on the spinner pictured below. Assume the spinner will not stop between choices.



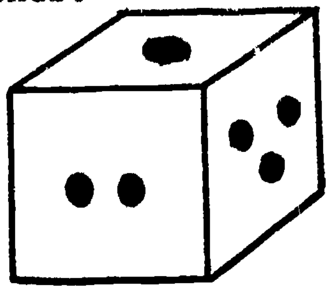
A. Experiment I: If you spin the arrow, what is the probability it will end up pointing at the circle?

Experiment II: The probability it will point to a right triangle?

Experiment III: The probability it will point to a triangle?

Experiment IV: The probability it will point to a rectangle?

B. Take a regular die. The six faces have dots, numbering one through six. Consider the following experiments.



Experiment I: (Example) If the die is rolled, what is the probability that the top face will have three dots?

P (three) = $\frac{1}{6}$

Experiment II: The probability of an even number showing? ______

Experiment III: The probability of an odd number showing? ______

What is the sum of the two probabilities in Experiment II and Experiment III? ______
What does this sum tell you? ______

Experiment IV: What is the probability of not rolling a six? ______

ACTIVITIES

1. A sample of fifty automobile tires of a particular brand were tested for mileage. The chart below shows the results.

Number of Tires	Mileage Before Defects or Blow Out Occurred
10	25,000
5	20,000
15	18,000
2	16,000
12	12,000
5	5,000
1	1,000

A. Based only on this information, what is the probability you would buy this brand of tire and get 25,000 miles? ______

B. The probability you'd get a tire that would go more than 12,000 miles? ______

C. The probability you'd get less than 20,000 miles? ______

D. Based on this information, can you arrive at a reasonable guarantee on mileage for these tires? ______

2. A lunch counter manager of a large department store decided to have a sale which involved a game. Pictures of bananas were made up and taped along the back wall. Prices ranging from 0 to 43¢ were written on the back of each banana. The customer could pick a picture of a banana and the price on the back was the cost of a banana split. The manager made the list below for his information.

30 pictures - 43¢	10 pictures - 20¢
18 pictures - 42¢	5 pictures - 9¢
20 pictures - 41¢	5 pictures - 2¢
19 pictures - 40¢	5 pictures - 1¢
15 pictures - 34¢	5 pictures - 0¢

If you pick a banana, what is the probability

A. your price will be 43¢? ________

B. your price will be 0¢? ________

C. your price will be more than 40¢? ________

D. your price will be less than 20¢? ________

E. If the store did not use this type sale, what would they have to charge for each banana split in order to make the same amount of money? ________

3. For advertising purposes, a large store asked each customer to place their name on a slip of paper and put it in a large box. A drawing was held at the end of the month. Below are listed the prizes and the number of customers that registered.

PRIZES: 1st - First two names drawn -- a color television for each person. (No more than <u>one</u> prize per person is allowed.)

2nd - Next four names -- a transistor radio for each person.

3rd - Next four names -- a free pass to a movie.

REGISTRATIONS: 6,216 customers

A. If you are one of the customers, what is the probability of your winning a prize? ________

B. What is the probability of your winning a color television? ______

C. What is the probability of your winning a radio <u>or</u> a free pass to a movie? ________

D. If one-third of the customers are men, what is the probability a man will win? ________ A woman? ________ A male or female (one or the other)? ________
Neither a male <u>nor</u> female will win? ________

4. "A man claims to have seen the sun rise <u>at</u> <u>least</u> fifteen times in one day."
Can a probability be associated with this claim?
<u>Think</u> carefully before you commit yourself to an answer.

<u>COUNTING OUTCOMES</u>

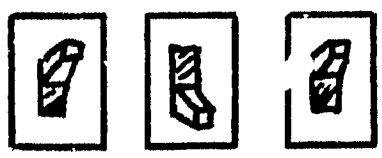
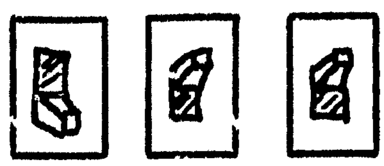
Many times it takes thought to figure the total number of different outcomes an experiment can have.

For example, say there are three light switches wired to one light such that two certain switches must be pushed up together before the light will burn. You do <u>not</u> know which two to push up. If you try <u>two</u>, what is the probability you are right? ________

First, we must determine how many total outcomes are possible. We already know only <u>one</u> of these will work. Then:

$$\text{Probability (light burning)} = \frac{\qquad}{\text{total possible outcomes}}$$

Let's make a chart of the possibilities.

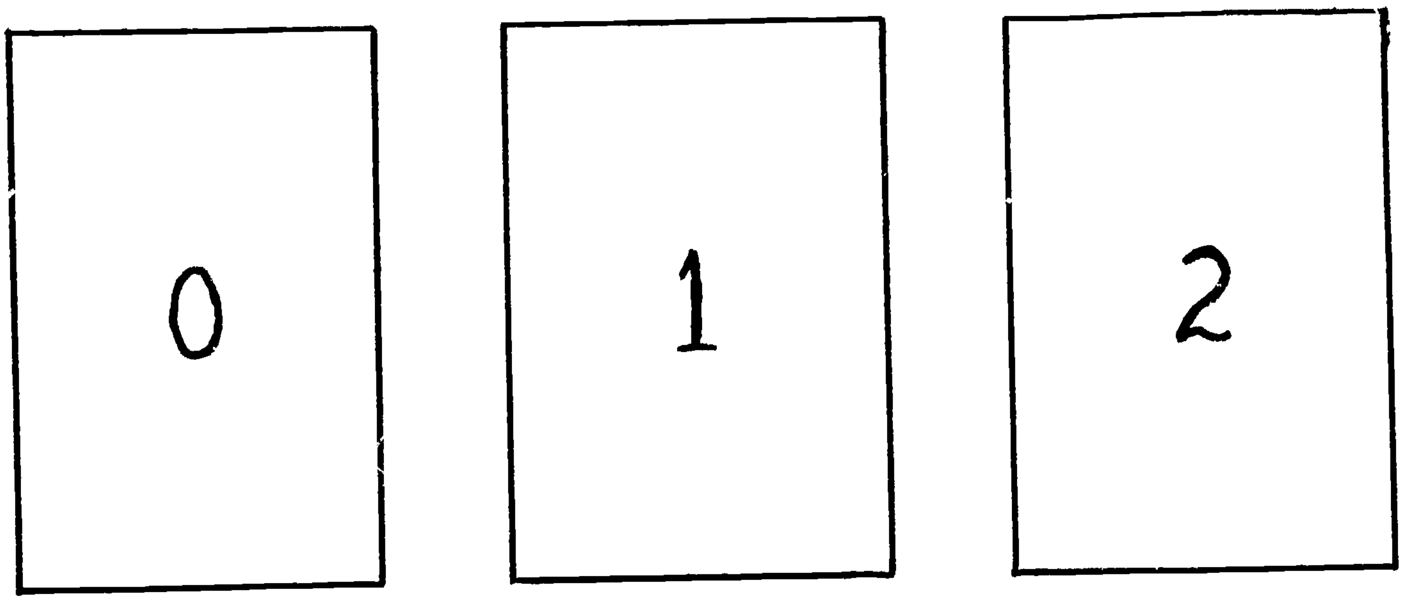
<u>Possibilities</u>			<u>Condition</u>
1.	ON OFF		1 and 2 on, 3 off
2.	ON OFF		1 and 3 on, 2 off
3.	ON OFF		2 and 3 on, 1 off

Are these all of the possible outcomes? The answer is YES.

Then our probability is: $P(\text{light on}) = \frac{1}{3}$

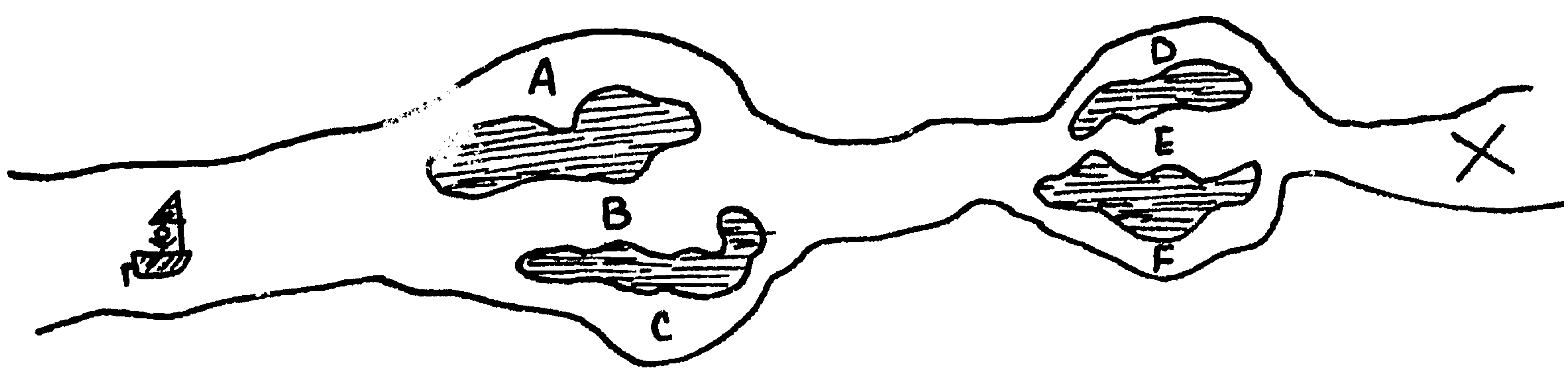
<u>Experiment I</u>: Cut out small cards and place a different number, 0 through 2, on each card. Put them face down and mix them up. Turn each up, placing them in a row.

What is the probability you will turn them up in order? (See picture below.)



<u>Experiment II</u>: You are going down river in a boat and plan to swim at the place marked with an X. The river has three branches, then comes back together, then three more branches that run back together. The drawing below shows the idea.

How many different routes can you follow to get to the place you'd like to swim? The different routes are marked.



If a boatload of friends are coming later, what is the probability they will follow the same route you did? ________

Make a table that lists the total possible outcomes (possibilities). The table is started below. Can you complete it and then answer the question above?

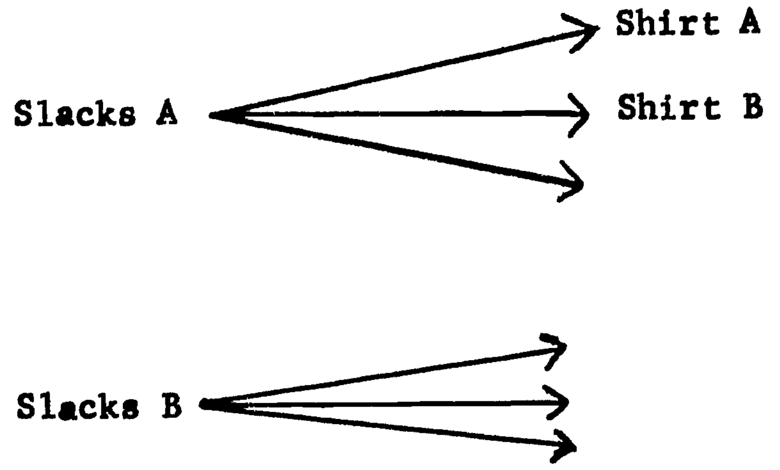
<u>ROUTES</u>

1. Through branch A and then through branch D (called AD).
2. Through branch A and then through branch E (called AE).

<u>Experiment III</u>: Suppose a boy has <u>three</u> shirts of different colors and <u>two</u> pair of slacks. If wearing a different shirt with the same slacks is considered a different way of dressing, how many different ways can he dress? ________

Complete the drawing below before you answer.

<u>Probability Tree</u>



<u>Different Ways</u>

<u>Slacks A</u>, <u>Shirt A</u>
<u>Slacks A</u>, <u>Shirt B</u>
________, ________
________, ________
________, ________
________, ________

Do you count <u>6</u>?

<u>Experiment IV</u>: From the information in experiment III, can you figure how many different ways of dressing he could choose from if he had four pair of slacks and four shirts? ________

Make a tree drawing to show your answer. (Probability Tree)

Mutually Exclusive Outcomes

If one outcome occurs, it can possibly exclude the possibility of certain other outcomes occurring. In other words, they cannot occur. To illustrate this, say you toss a coin and it comes up heads. Does this exclude the possibility of tails? These two outcomes are called mutually exclusive outcomes.

Some of the statements below are definitely false. They indicate that two events occurred that were mutually exclusive. Can you pick them out? Think before you answer.

1. "Our team scored the most points in the basketball game, but we lost." ___

2. "While hunting in the area of Belle Glade, I noticed the sun was just coming up. I checked my watch and it was 11:30." _____

3. "We were driving sixty miles per hour and passed a train which was going ninety miles per hour." _____

4. "Beth is celebrating her fourth birthday. She is sixteen years old." ____

5. "This was my first summer trip to Key West, and it snowed the first day." _____

Simply stated, two events are mutually exclusive if they cannot occur together.

Independent Outcomes

Now that you can determine whether outcomes are mutually exclusive, suppose we consider independent outcomes. Independent outcomes (or events) are events such that the occurrence or non-occurrence of one is not affected by the occurrence or non-occurrence of others. This sounds complicated but is actually not that hard. Some examples of independent outcomes are given below.

Toss a coin two times. The outcomes are independent. This is to say that regardless of whether you get a heads or tails the first toss, it has no effect on getting a heads or tails the second toss.

If you have tossed a coin eight times and have gotten heads each time, are you more likely to get a tails than heads on the next toss? _____

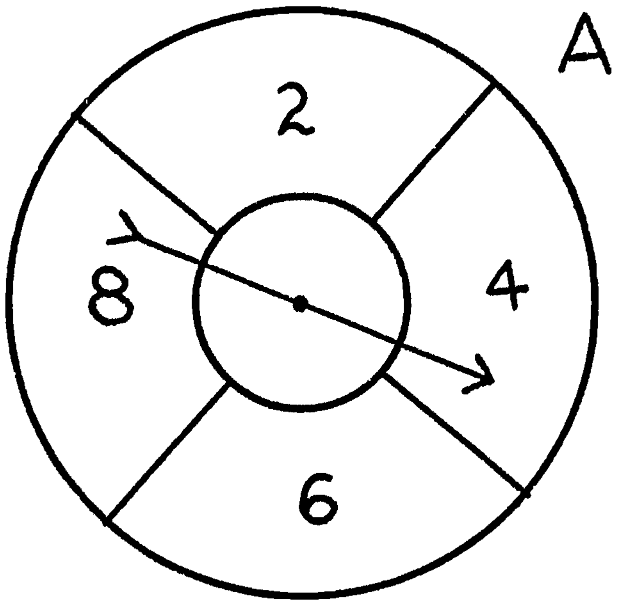
Your answer should be NO. Consider each toss individually. Heads or tails are the only two outcomes.

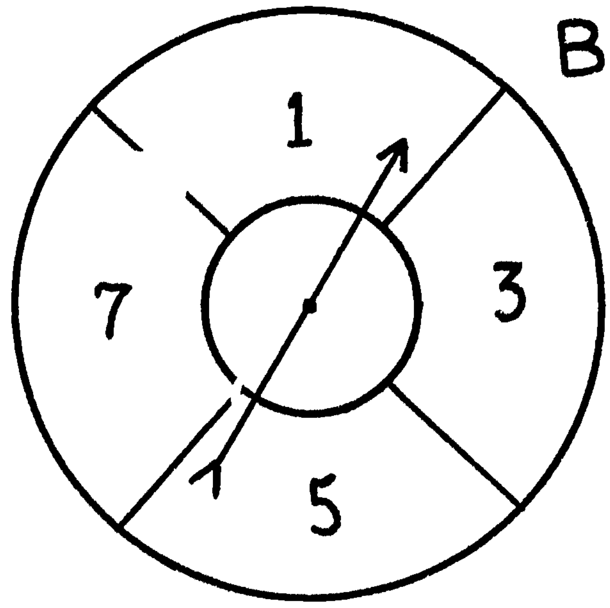
The probability of a heads on any given toss is: $P\ (\text{heads}) = \frac{1}{2}$

For the next toss, the probability is the same. If you tossed the coin 1,000 times, this probability would not change.

If the one outcome does not change the probability of a second outcome, the outcomes are independent.

Below are two spinners. Conduct the following probability experiments with these spinners. Refer to them as A and B.





1. What is the probability of spinning A and getting an odd number?_____

2. What is the probability of spinning A and getting a number that is divisible by 4 (zero remainder)? _____

3. Let's say you spin A, then B. Will the result you get from B be independent of the result you got from A? _____

4. Suppose you spin A and then B. The different possible outcomes are listed.

Possible Different Outcomes

Spinner A	Spinner B
2	1
2	3
2	5
2	7
4	1
4	3
4	5
4	7
6	1
6	3
6	5
6	7
8	1
8	3
8	5
8	7

Do you observe <u>16</u> possible different outcomes by spinning both A and B? Spinner A has 4 possible outcomes and spinner B has 4 possible outcomes which are independent of A. Then an experiment involving A and B has:

<u>4 x 4 = 16 different outcomes</u>

<u>Multiplication Property</u>: In general terms, if one operation can occur in "m" ways, and a second independent operation can occur in "n" ways, an experiment involving both has:

<u>m x n different outcomes</u>

Using the possible outcomes listed above, if you spin A and B, what is the probability

4- a. you'll get two numbers that <u>add</u> to <u>7</u>? _______

4- b. you'll get a <u>2</u> on spinner A and a <u>7</u> on spinner B? _______

4- c. you'll get two numbers that add to 15? _______

4- d. both numbers will be prime numbers? _______

4- e. both numbers will be even? _______

4- f. the sum of the two numbers will be even? _______

4- g. the sum of the two numbers will be odd? _______

4- h. the product of the two numbers will be odd? _______

<u>ACTIVITIES</u>

Use our multiplication property to answer the following problems. The first is an example.

Example: A football team has <u>3</u> quaterbacks and <u>2</u> centers. How many different possible choices does the coach have to start a game?

3 x 2 = <u>6</u>

To show this, call the quaterbacks A, B, C and the centers M and N.

<u>Different Possible Choices</u>

A M
A N
B M
B N
C M
C N

1. In a restaurant a man has a choice of 3 meats and 4 vegetables. He only wants 1 meat and 1 vegetable. How many different possible outcomes does he have to choose from? ______

2. If three coins are tossed at one time, how many outcomes are possible? ______
Complete the list to check your answer.

Possible Outcomes

Coin I	Coin II	Coin III
head	head	head
head	tail	head
______	______	______
______	______	______
______	______	______
______	______	______
______	______	______

Use the outcomes listed above to answer the following questions.

2-a. What is the probability of getting 3 heads? ______

2-b. The probability of getting 1 head and 2 tails? ______

2-c. The probability that all three will come up tails? ______

3. A girl has 6 skirts and 5 blouses. If she can wear any combination, a skirt with a blouse, how many different outfits can she wear? ______